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Shot Noise in a Spin-Diode Geometry

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Abstract We apply the master equation technique to calculate shot noise in a system composed of single level quantum dot attached to a normal metal lead and to a ferromagnetic lead (NM-QD-FM). It is known that this system operates as a spin-diode, giving unpolarized currents for forward bias and polarized current for reverse bias. This effect is observed when only one electron can tunnel at a time through the dot, due to the strong intradot Coulomb interaction. We find that the shot noise also presents a signature of this spin-diode effect, with a super-Poissonian shot noise for forward and a sub-Poissonian shot noise for reverse bias voltages. The shot noise thus can provide further experimental evidence of the spin-rectification in the NM-QD-FM geometry.

Keywords Spintronics · Shot noise · Quantum dot

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1 Introduction

Shot noise is an unavoidable temporal fluctuation of the current in mesoscopic devices that comes from the granularity of the current carriers [1]. In contrast to other noise sources (e.g. thermal noise) the shot noise contains a wealth of additional information not obtained via the average current. For example, the shot noise revealed the existence of quasi-particle with charge $e/3$ in the fractional quantum Hall effect [2]. In the context of spintronics [3], shot noise is a powerful quantity to give further information about spin-dependent phenomena [4, 5]. For instance, strong suppression of the shot noise in carbon nanotubes attached to one ferromagnetic lead and one normal metal lead provides additional information regarding spin accumulation and spin correlation effects [6].

The noise power spectrum, that contains both shot noise and thermal noise, is given by the Fourier transform of the current–current correlation function $\langle I_\eta(t)I_{\eta'}(0)\rangle$, i.e.,

$$S_{\eta\eta'}(\omega) = 2\int_{-\infty}^{\infty} dt\, e^{i\omega t}\big[\big\langle I_\eta(t)I_{\eta'}(0)\big\rangle - \langle I\rangle^2\big], \tag{1}$$

where $\langle I\rangle$ is the average current in the long-time range, and $\langle\ldots\rangle$ denotes an ensemble average. The indices η and η' label the electrodes: $\eta = L$ for the left and $\eta = R$ for the right electrode.

With the formulation developed in Ref. [7] we calculate the zero-frequency limit of (1) for a system composed of a single level quantum dot attached to a left normal metal lead and to a right ferromagnetic lead (NM-QD-FM). Our system in thus asymmetric in the sense that only one electrode is ferromagnetic. It is already known that this particular geometry gives a spin-diode effect–unpolarized current for forward bias and polarized current for reverse bias–when the system operates in the single occupancy regime [8]. This effect was recently confirmed experimentally [9]. In what follows we will apply an external source-drain bias voltage and look at the transport in the nonlinear regime. For positive voltages (forward) the electrons flow from the NM to the FM electrode (left to right), while for negative biases (reverse) the flow occurs from the FM to the NM lead (right to left). In this context, the shot noise can be a useful quantity to be measured, thus providing additional/complementary information about the physics underlying the spin-diode effect. Here in particular, we show a correlation between current polarization and shot noise. We found that the shot noise is super- Poissonian when the current polarization is zero for forward biases and sub-Poissonian for polarized current in reverse bias voltages.

2 Transport Formulation

The quantum transport calculation is developed in the framework of the master equation technique [7], where both the current and the noise are related to the transition matrix

$$\mathbf{M} = \begin{pmatrix} -(\Gamma_\uparrow^+ + \Gamma_\downarrow^+) & \Gamma_\uparrow^- & \Gamma_\downarrow^- \\ \Gamma_\uparrow^+ & -\Gamma_\uparrow^- & 0 \\ \Gamma_\downarrow^+ & 0 & -\Gamma_\downarrow^- \end{pmatrix}, \tag{2}$$

where the tunneling rates are

$$\Gamma_\sigma^+ = \Gamma_L^\sigma f_L + \Gamma_R^\sigma f_R, \tag{3}$$

$$\Gamma_\sigma^- = \Gamma_L^\sigma (1 - f_L) + \Gamma_R^\sigma (1 - f_R). \tag{4}$$

Here Γ_η^σ gives the tunneling rates between lead η and the dot for electrons with spin σ. This quantity is related to the spin-resolved density of states of lead η [10],

$$\Gamma_\sigma^L = \Gamma_0^L [1 + \sigma' p_L], \tag{5}$$

$$\Gamma_\sigma^R = \Gamma_0^R [1 \pm \sigma' p_R], \tag{6}$$

with $\sigma' = +$ or $-$ for spin $\uparrow$ or $\downarrow$, respectively. The parameters Γ_0^L and Γ_0^R give the leads-to-dot coupling strength and p_η accounts for the lead polarization. The signs $\pm$ in Γ_σ^R are for parallel (+) and antiparallel (−) alignments. In particular in what follows, we consider $p_L = 0$ which corresponds to the spin-diode geometry [8]. In (3)–(4) the quantities f_L and f_R are the Fermi distribution functions of the left (L) and right (R) leads evaluated at the dot level ϵ_d,

$$f_\eta(\epsilon_d) = \frac{1}{e^{(\epsilon_d - \mu_\eta)/(k_B T)} + 1}, \tag{7}$$

where $\epsilon_d = \epsilon^0 - eV/2$, $\mu_L = E_F = 0$ and $\mu_R = E_F - eV$, with ϵ^0 being the dot level without bias, μ_L and μ_R the left and right chemical potentials, E_F the Fermi level of the electrodes, e the electron charge ($e > 0$) and V the bias voltage. Note that the left chemical potential is taken at zero (energy reference), while the right one increases linearly with eV. Following a linear drop of bias voltage from the left to the right side of the junction we have assumed a variation of $eV/2$ for the dot level. The resonances will occur whenever $\epsilon_d = \mu_L$ ($eV > 0$) or $\epsilon_d = \mu_R$ ($eV < 0$). In particular, for $\epsilon_d^0 = 0.5$ meV (value adopted in the simulations) the resonant tunneling will start at $eV = 1$ meV. In the present work we consider a bias range in which the charging energy U is not overpassed, i.e., $\epsilon_d + U$ remains above the left and the right chemical potentials. So the transport is carried only by a single electron via the channel ϵ_d. This is the Coulomb blockade in the present system, since one electron in the dot forbids a second electron to tunnel into the dot. Observe that the Coulomb blockade here does not suppress to zero the entire current.

The density matrix elements evolve according to

$$\frac{d\boldsymbol{\rho}(t)}{dt} = \mathbf{M}\boldsymbol{\rho}(t), \tag{8}$$

where $\boldsymbol{\rho} = (\rho_0, \rho_\uparrow, \rho_\downarrow)^T$, with ρ_0 being the probability of the dot being empty and $\rho_{\uparrow(\downarrow)}$ the probability of the dot being occupied with a single electron of spin $\uparrow$ ($\downarrow$). In the present study we consider only the single occupancy regime, so the double occupancy probability is equal zero. We are particularly interested in the stationary regime, i.e., $d\boldsymbol{\rho}(t)/dt = 0$. This means that we need to determine an eigenvector $\boldsymbol{\rho}^{(0)}$ corresponding to a zero eigenvalue of the matrix $\mathbf{M}$, i.e.,

$$\mathbf{M}\boldsymbol{\rho}^{(0)} = 0. \tag{9}$$

With $\boldsymbol{\rho}^{(0)}$ we can calculate the stationary current in lead η and spin σ from

$$I_\eta^\sigma = e \sum_\nu \left[\boldsymbol{\Gamma}_\eta^\sigma \boldsymbol{\rho}^{(0)}\right]_\nu, \tag{10}$$

where

$$\boldsymbol{\Gamma}_\eta^\sigma = \begin{pmatrix} 0 & \delta_{\sigma\uparrow}\Gamma_{\eta\uparrow}^- & \delta_{\sigma\downarrow}\Gamma_{\eta\downarrow}^- \\ \delta_{\sigma\uparrow}\Gamma_{\eta\uparrow}^+ & 0 & 0 \\ -\delta_{\sigma\downarrow}\Gamma_{\eta\downarrow}^+ & 0 & 0 \end{pmatrix}. \tag{11}$$

The noise calculation is a bit more trick and it involves two parts. The first one is the Schottky term [12, 13]

$$S_{\eta,\sigma}^{Sch} = 2e^2 \sum_\nu \left|\left[\boldsymbol{\Gamma}_\eta^\sigma \boldsymbol{\rho}^{(0)}\right]_\nu\right|, \tag{12}$$

that gives the self-correlation contribution to the noise. This is related to the tunneling of electrons through the barrier in the η side. The second term of the noise comes from the correlation of the current in different times, and it is given by [13]

$$S^c_{\eta\eta'}(\omega) = 2e^2 \sum_{\nu,\lambda\neq 0} \left(\frac{[\mathbf{\Gamma}_\eta \mathbf{P}_\lambda \mathbf{\Gamma}_{\eta'} \boldsymbol{\rho}^{(0)}]_\nu}{-i\omega - \lambda} + \frac{[\mathbf{\Gamma}_{\eta'} \mathbf{P}_\lambda \mathbf{\Gamma}_\eta \boldsymbol{\rho}^{(0)}]_\nu}{i\omega - \lambda} \right) \tag{13}$$

where $\mathbf{\Gamma}_\eta = \mathbf{\Gamma}^\uparrow_\eta + \mathbf{\Gamma}^\downarrow_\eta$ and $\mathbf{P}_\lambda$ is a projector operator defined as

$$\mathbf{P}_\lambda = \mathbf{S}\mathbf{E}^{(nn)}\mathbf{S}^{-1}, \tag{14}$$

with $\mathbf{S}$ being constructed with the eigenvectors of $\mathbf{M}$ on its columns and $\mathbf{E}^{(nn)}$ is a matrix with an element 1 in the position nn and zero elsewhere. The label n corresponds to the n-th eigenvalue. The total noise is the sum $S_{\eta\eta'}(\omega) = \delta_{\eta\eta'} S^{Sch}_{\eta,\sigma} + S^c_{\eta\eta'}(\omega)$. We should note that the present transport problem (current and noise) is thus equivalent to an eigenvalue–eigenvector problem (see (9)). As a summary of the methodology, we have essentially to calculate the eigenvector $\boldsymbol{\rho}^{(0)}$ of $\mathbf{M}$ corresponding to the zero eigenvalue, then use it in (10) to determine the spin-resolved currents. We also obtain all the eigenvalues of $\mathbf{M}$ to use, together with $\boldsymbol{\rho}^{(0)}$, in (12)–(13) to find the noise.

3 Results and Discussions

Figure 1(a) shows the current polarization, $\wp = (I^\uparrow_L - I^\downarrow_L)/(I^\uparrow_L + I^\downarrow_L)$ as a function of the bias voltage eV for different polarizations of the FM electrode. The bias range in Fig. 1(a) ($-3 <$ eV < 3 meV) allows only single occupancy, i.e., the dot level ϵ_d attains resonance ($\epsilon_d < \mu_L$ for $eV > 0$ and $\epsilon_d < \mu_R$ for eV < 0), while $\epsilon_d + U$ remains above both μ_L and μ_R. Since the channel $\epsilon_d + U$ stays above the chemical potentials only one electron per time can enter the dot. This corresponds to a Coulomb blockade regime. Note that a second electron is forbidden to tunnel into the dot because of the Coulomb repulsion of the single electron already in it. In particular, for the numerical parameters adopted here, the resonance $\epsilon_d = \mu_L$ $(eV > 0)$ or $\epsilon_d = \mu_R$ $(eV < 0)$ is achieved at $eV = \pm 1$ meV. The result in panel (a) is the already known spin-diode effect [8, 9], that gives an unpolarized current for positive bias, while a polarized current for negative bias voltage. The plateau of $\wp$ for eV < 0 is equal to p. We should emphasize that the suppression of the current polarization ($\wp = 0$) for positive biases happens only in the single occupancy regime ($\epsilon_d + U > \mu_L, \mu_R$). When two electrons with opposite spins can simultaneously tunnel into the dot, $\wp$ attains a nonzero plateau for both positive and negative voltages.

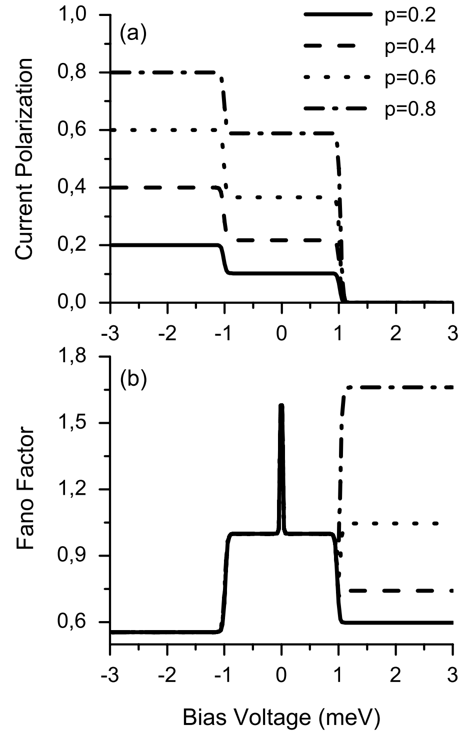

Fig. 1 Current polarization and Fano factor against bias voltage for different lead polarizations: $p = 0.2$, 0.4, 0.6 and 0.8. The current polarization is independent of p and equal zero for positive bias and it attains a relative high value for negative biases. In contrast, the Fano factor reaches super-Poissonian values for positive bias voltages as p increases and it is sub-Poissonian for negative voltages. The super-Poissonian Fano factor can be used as an evidence of the spin-diode effect in experimental measurements. Parameters: $\epsilon^0 = 0.5$ meV, $U = 1$ meV, $\Gamma_0 = 10$ μeV, $k_B T = \Gamma^0$

Since measurements of the current polarization is in general difficult to be performed, additional transport information in this spin-diode geometry is desirable. Figure 1(b) shows the Fano factor, $F = S_{LL}/(2eI_L)$, against eV for the same set of p values presented in panel (a). Here S_{LL} is the zero-frequency shot noise. Interestingly, in the bias range in which $\wp = 0$, the Fano ratio can attain values well above one when p increases. On the other hand, for negative voltages, F assumes a constant value close to 0.6 that is independent of p. In particular for $p = 0$ the left and right Fano factor plateaus become the same, given by [11]

$$F = 1 - \frac{4\Gamma^L_0 \Gamma^R_0}{(\Gamma^R_0 + 2\Gamma^L_0)^2}. \tag{15}$$

For positive bias voltages, electrons with spin up and down tunnel with equal rates into the dot coming from the emitter lead (NM electrode), however the spin up ones tunnel to the collector lead (FM electrode) faster than the spin down ones. This gives rise to the so-called dynamical spin blockade in which the spin that tunnels with lower rate governs the tunneling events of the other spin component. Consequently a bunching of electron tunneling takes place, thus resulting in $F > 1$ [12].[1] The peak in the Fano factor at $eV = 0$ is due to thermal noise.

4 Conclusions

We have applied the master equation technique to calculate the spin-dependent current and shot noise in a system composed of a quantum dot coupled to a nonmagnetic and to a ferromagnetic electrode (NM-QD-FM). In the bias range ($eV > 0$) in which the current polarization is zero, due to Pauli and Coulomb correlations, the Fano factor reveals super-Poissonian shot noise ($F > 1$) for large magnetization degree p. In contrast, for $eV < 0$ while the current polarization changes with p, the Fano factor attains a plateau independent of p. This behavior of the Fano factor can provide further insights regarding the physical mechanism underlying the spin-diode effect and better guide future experimental realizations of this junction (NM-QD-FM).

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[1] Super-Poissonian noise can also be observed in a system of a quantum dot attached to two ferromagnetic leads in the parallel configuration, see Ref. [13].